1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

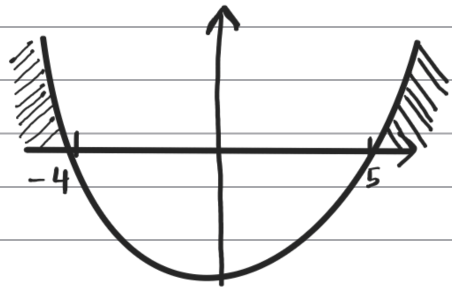
writing your answer in set notation.

(3)

$x^2 - x > 20$

$x^2 - x - 20 > 0$

$(x+4)(x-5) > 0$



1

Find the roots using quadratic equation

$x + 4 = 0 \qquad x - 5 = 0$

$x = -4 \qquad x = 5$

Roots $= -4$ and 5 1

From here, you can then write the answer in set notation

1

$\{x : x < -4\} \cup \{x : x > 5\}$

DO NOT WRITE IN THIS AREA

2. (a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

(a) Since x and y are both positive, their square roots are real, and so we can use:

$$(\sqrt{x} - \sqrt{y})^2 \geq 0$$

$$(\sqrt{x} - \sqrt{y})(\sqrt{x} - \sqrt{y}) \geq 0$$

$$x - 2\sqrt{x}\sqrt{y} + y \geq 0$$

$$x - 2\sqrt{x}\sqrt{y} + y \geq 0$$

$$x + y \geq 2\sqrt{x}\sqrt{y}$$

$$2\sqrt{x}\sqrt{y} \leq x + y$$

$$\therefore \sqrt{x}\sqrt{y} \leq \frac{x+y}{2}$$

$$\therefore \boxed{\sqrt{xy} \leq \frac{x+y}{2}}$$

(b) If $x = -2$ and $y = -3$, then:

$LHS = \sqrt{(-2)(-3)} = \sqrt{6}$

$RHS = \dfrac{-2+(-3)}{2} = \dfrac{-2-3}{2} = -\dfrac{5}{2}$

so $\sqrt{xy} > \dfrac{x+y}{2}$ in this case

3. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

i) $x^2 - 8x + 17 = (x-4)^2 + 17 - 16$

$= (x-4)^2 + 1$

$(x-4)^2$ is always greater than 0

$\therefore x^2 - 8x + 17 > 0$ for all real values of x

ii) for $n=1$, $(1+3)^2 = 16$ $\quad 1^2 = 1$

$16 > 1$ $\therefore$ true for $n=1$

for $n=-3$ $\quad (-3+3)^2 = 0$ $\quad (-3)^2 = 9$

$0 < 9$ $\therefore$ false for $n=-3$

$\therefore$ statement is sometimes true

4.

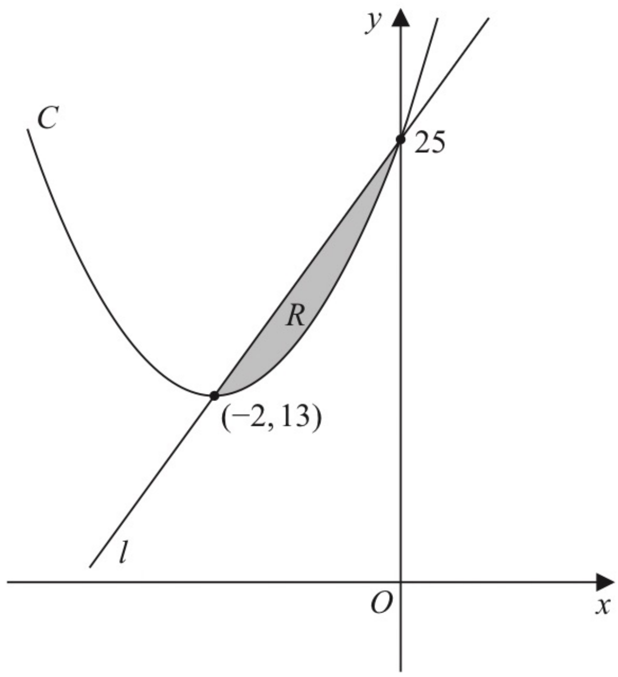


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define R.

(5)

a) $l: y = mx + c$, $c = 25$ (y-intercept on graph)

We will use the point $(-2,13)$ to work out m.

$(-2, 13)$: $13 = -2m + 25 \Rightarrow 2m = 12$ 1

$\Rightarrow m = 6 \Rightarrow l: y = 6x + 25$ 1

$\Rightarrow f(x) = a(x+2)^2 + 13$

$\Rightarrow (0, 25)$: $25 = 4a + 13 \Rightarrow 4a = 12$, thus $a = 3$ 1

$\Rightarrow C: y = 3(x+2)^2 + 13$ 1

$\Rightarrow 3(x+2)^2 + 13 < y < 6x + 25$ 1